System Stability in Wood Truss Assemblies during Construction

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Learn about emerging research on bracing strategies for system stability.

Authors' Note: This paper is a modification of a paper presented at the World Conference on Timber Engineering in August 2006 in Portland, OR. We appreciate the contributions of Professor Dan Wheat from the University of Texas-Austin.

$$P_{cr} = \frac{\pi^2 EI}{(K_e L)^2}$$

Equation 1. Euler's Buckling Formula.

at a glance

- Euler's buckling formula dates back to 1757; we still use it today to determine bracing.
- With the creation of an analysis program that more accurately predicts the behavior of a truss system, bracing designs can be examined.
- Given that relatively few material properties were measured, the SAWFT predictions are surprisingly close to the instability loads obtained from the tests.
- This work shows promise as a first step toward development of robust structural analysis algorithms for developing new design procedures for wood structural system stability.

Summary

Most framers and non-engineering construction people have a practical understanding of gravity loads and the basic resistance of these loads exhibited primarily as tension and compression in the building components that support them. Compression, however, can lead to crushing in some instances and buckling in others. By definition and theory, buckling is a sudden event which can occur at loads far below those necessary to cause crushing. As a result, it is a failure mode that can be unexpected for those unfamiliar with the concept.

Trusses must be braced during construction and in service to prevent buckling. The current approach for determining bracing requirements is to evaluate individual truss members independently of other members and trusses they may be attached to. Since metal plate connected wood trusses are primarily used as part of a structural system of repetitive members, a more realistic approach would be to evaluate bracing requirements by considering the behavior of the entire roof or floor system. Computational techniques currently exist to extend the structural analysis of a wood truss system to consider stability. This article reviews several computational approaches and demonstrates the application of one method to the prediction of system stability with different bracing strategies. Once prediction of system stability is shown to be accurate, the method can also be used to examine the forces that occur within the braces.

Introduction

If trusses were restrained and braced according to the recommendations provided in the *Building Component Safety Information booklet, BCSI, Guide to Good Practice for Handling, Installing, Restraining & Bracing of Metal Plate Connected Wood Trusses,* there would not be much to write about here. As truss installers look for the most efficient ways to install trusses, industry recommendations are often modified or ignored, sometimes resulting in performance problems and/or truss collapse. This article discusses the results of some preliminary research involving stability calculations for truss systems. The computations suggested here are not quite ready for everyday design, but perhaps they will be in the near future.

The rules upon which current temporary and permanent bracing recommendations are based are fairly simple and rely on basic engineering concepts of triangulation of members, Euler's buckling formula, and the "2% Rule." Euler's buckling formula (see Equation 1) dates back to 1757 and yes, 250 years later, we still use it to determine bracing! This formula is used to calculate the theoretical elastic buckling load of a compression member based on the modulus of elasticity, E, unbraced length of the subject member, L, moment of inertia, I, and effective length factor, Ke, for the member (see sidebar for the derivation of the 2% Rule).

If the buckling load is exceeded, either the size and/or grade of the member must be increased, or restraint and bracing must be added to reduce the unbraced length. The force required to prevent buckling is typically assumed to equal two percent (thus the 2% Rule) of the compression force in the member. Using this information the size and connection requirements for the lateral restraint and diagonal bracing can be determined.

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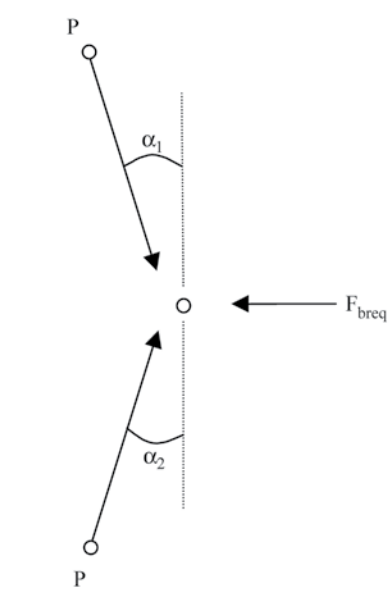
The 2% Rule[1]

The 2% Rule is a strength model used to determine the amount of load required to stabilize a column against buckling. Designers primarily use strength models during design due to the simplicity of the calculations. In the derivation of the 2% Rule, the column is assumed to be pinned at each end and at the center restraint location. Throop (1947) explained where the 2% Rule originated and Nair (1992) and Waltz (1998) illustrated the use of a force balance in the development of the 2% Rule. The column is assumed to be one-inch out of plumb for an assumed story height of 100 inches (Throop, 1947). A compression chord restrained at the center of its span will have a force of 1/100 above and below the restraint as depicted in Figure A below. A force balance for the free body diagram at the restraint is the basis for the 2% Rule (Nair, 1992).

[1] Underwood, Catherine R. 2000. *Bracing Design for a MPC Wood Truss Member with Numerous Brace Locations.* MS Thesis, Virginia Polytechnic Institute and State University, Blacksburg, VA.



where:

P = column load (pounds)

F_{br} = restraint force

α_1 = angle between brace and vertical plane

α_2 = angle between brace and vertical plane

Force Balance

Assuming pins at column ends and at point of restraint attachment:

$$F_{br} = P \sin(\alpha_1) + P \sin(\alpha_2)$$

When α_1 and α_2 are small and equal, $\sin_\alpha$ approximately equals $\tan_\alpha$. From Throop (1947), $\tan_\alpha$ was assumed to be 1/100, therefore:

$$F_{br} \approx (1/100 + 1/100)\ P$$

$$F_{br} \approx 0.02P, \text{ or } 2\% \text{ or } P$$

Figure A. A free body diagram and a force balance depicting the origin of the 2% Rule as presented by Waltz (1998), Throop (1947) and Nair (1992).

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There is probably an inherent conservatism using this approach since the compression members in trusses are embedded within a system and the stability of the system cannot be readily reduced to the performance of a single member. This article presents a way to consider truss system stability and the design of bracing, by considering the system as a whole. Such approaches are computationally more complex, but the technical foundation already exists in the structural analysis methods currently used by the industry to analyze and design trusses.

Different Computational Approaches to Buckling

The standard linear stiffness analyses currently used by all the major software providers to design trusses assume in the calculations that the displacements and rotations are very small compared to the length of the truss members. As part of this approach, member forces are computed based on the undeformed geometry of the truss. Displacements that are one percent or less of the length of the wood members are considered small or infinitesimal and those that are larger are called finite displacements. By lifting the "small displacement" restriction in the computer software, buckling and system instability can also be determined but the analysis becomes more complex. The numerical prediction of buckling and structure instability falls under the branch of structural mechanics called geometric nonlinearity. When displacements (and rotations) are small (infinitesimal), structural analysis is a linear problem that is easily solved. When displacements (and rotations) are larger (finite), structural analysis becomes nonlinear and solutions are obtained by iteration.

There are a variety of structural analysis approaches that consider geometric nonlinearity. Given the importance of structural stability to the building component industry it is timely to see how these methods can be used to improve building component stability and bracing design. Recently we examined three methods that can be classified according to the assumptions concerning the magnitude of displacement as follows:

1. methods that address finite displacements but infinitesimal rotations,
2. methods that address finite displacements and finite rotations, and
3. methods that address the effect of finite displacements through the inclusion of a geometric stiffness matrix.

The first method has the advantage of being essentially the same as the current methods used in truss design software except analysis of a given truss configuration must be done iteratively. It simplifies the software and solution computations if the rotations are small even if the displacements are large. The main advantage of this method is simplicity, in that it involves doing more of the analysis that is currently employed for design. The primary disadvantage is that as the single truss or assembly becomes more complex, the number of iterations required to find the maximum load before buckling occurs can be impractically large. Thus while this method would require little change to current software, analyses would be too time-consuming even with high speed computers and in some cases may not even provide a solution.

The introduction of finite rotations in the second method requires a new set of stiffness coefficients that are significantly more complex than currently used in truss design. We have not implemented this method because it only presents an advantage over the first where the members undergo large rotations prior to reaching instability. Again iterations to update the deformed shape are necessary and, in many situations with complex systems, the number of iterations to identify the maximum load before instability is large.

The third method also offers simplicity and can take two forms. One form is referred to as a two-part linear analysis which, in its most common form, uses 1) elastic and 2) geometric stiffness matrices to represent the stiffness of the structure. The geometric stiffness matrix introduces nonlinear effects into the analysis more directly than in the two methods above. The nonlinear effect considers interrelationships such as those between bending and axial forces. For example, a column experiencing a lateral load will fail much sooner with a compressive load applied than if no lateral load is applied. The first form of the method is advantageous, as no iterations are necessary, but the disadvantages are incomplete structural information and for some problems inaccurate predictions of the load causing buckling. Therefore we have examined a second form of this method.

This second form is similar to the one just mentioned but involves tracking the load-displacement behavior of the structure. With this second approach, the loads are applied in steps as opposed to the full load all at once. The changing geometry of the truss is constantly updated according to the new displacements in each step. Instability occurs once the displacements become very large during a small load increment. This approach was chosen as it is the most robust, efficient and accurate candidate among the options listed above.

The geometric stiffness matrix used here was added to the elastic stiffness matrix for each member and incorporated in a truss analysis software called SAWFT (Structural Analysis of Wood Frames and Trusses) by Samiappan (2005) and the authors. These two matrices allow for changes in axial stiffness due to axial load, the interaction between bending stiffness and axial force, and the interaction between torsional stiffness and axial force. Instabilities associated with axial forces in compression members and torsion in bending members are included.

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Example Problems & Results

Single member examples: With the creation of an analysis program that more accurately predicts the behavior of a truss system, bracing designs can be examined. However, before attempting to analyze truss systems, it was first necessary to ensure the mathematical algorithms in the program were accurate. Two simple models were tested to check the programs accuracy. The two examples tested included an axially loaded column and a cantilever beam loaded transversely (see Figure 1). The theoretical critical buckling loads for these problems are derived from classical buckling theory, Equation 1 for the column (see page 94) and Equation 2 below for the beam.

$$P_{cr} = \frac{4.013}{L^2} \cdot \sqrt{E \cdot I_y \cdot G \cdot J}$$

Equation 2.

Where I_y is the minor axis moment of inertia, J as the section torsional constant, G is the shear modulus and the other quantities are as previously defined.

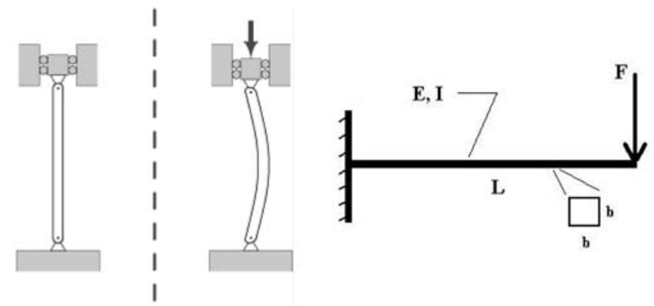

Figure 1. Simple column and cantilever beam.

Our predictions were computed as loads at which the members lose stability. We will refer to the loss of stability as the *capacity,* differentiating it from very detailed theoretical definitions of buckling. In each case the subject member consisted of a nominal 2x4 member with a 10-ft length. The modulus of elasticity (E) was 1.80 million psi with a shear modulus (G) equal to 1/16th of E, or 112,500 psi. The results are shown in Table 1 (see facing page) and reveal that while accuracy increases as expected with more elements, even a relatively few elements provide a satisfactory solution compared to the theoretical value.

Wood column assembly: With a theoretically verified algorithm, we proceeded to analyze a simple wood assembly consisting of seven parallel 2x4 members each 12' in length, spaced 2' apart (see Figure 2). Using the flatwise E and initial curvature data from Waltz et al. for Douglas-fir 2x4s, Monte Carlo simulation (a process where different values of E and curvatures are randomly selected from a preformed list of possibilities) of 50 of these assemblies was conducted for the following cases:

1.) Straight members with randomly selected E and no mid-height restraint.
2. Randomly selected E and randomly selected initial curvature with no mid-height restraint.
3. Randomly selected E and randomly selected initial curvature with 2x4 mid-height lateral restraint connecting the members but unattached to a boundary.
4. Same as 3 but the lateral restraint is attached to a flexible support representative of the stiffness of a nailed connection.

	10-ft Column Axial Load (lb)	% error from classical theory	10-ft Beam End Load (lb)	% error from classical theory
Theoretical	1215	NA	211	NA
Nonlinear [2 elements]	1223	0.7 %	225	6.6 %
Nonlinear [3 elements]	1216	0.1 %	216	2.4 %
Nonlinear [4 elements]	1215	0.0 %	214	1.4 %

Table 1. Instability loads for single member theoretical verification examples.

	Case 1	Case 2	Case 3	Case 4
Average Capacity (lbs)	3800	3330	4950	14200
% Difference Relative to Case 1	0	-12	30	274

Table 2. Average system capacities for Cases 1, 2, 3 and 4, and percent differences relative to Case 1.

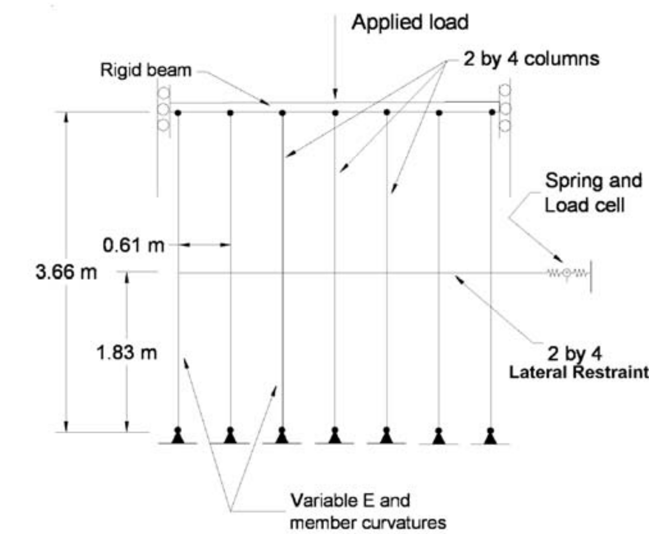

Figure 2. Simple seven member wood column assembly for stability.

The results, as expected, show that systems with no restraint or bracing (i.e., Case 1 and Case 2), reach their capacity (i.e., lose stability) when the member with the lowest E reaches its Euler buckling load.

Table 2 provides a summary of the average system capacities for Cases 1-4 and the percent differences in capacities relative to Case 1. Review of these data indicates that by including random curvature or lack of straightness and no restraint or bracing (i.e., Case 2), the average capacity of the system is reduced by approximately 12 percent. The average capacity for Case 3 showed a 30 percent increase over Case 1 (straight member) since random member curvatures will bias individual members to displace in different directions and, in connecting them together, they act as a system. By attaching the lateral restraint to a support corresponding to a nailed con-

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Truss Testing	Capacity (in lbs) Laboratory Test Load	Euler Buckling Load	SAWFT Load	% Error Euler Buckling Equation	SAWFT
Test I: Single truss	1820	1248	1730	-31	-5
Test II: Three truss assembly connected with 2x4 short member lateral restraint	3200	5256	3210	64	0
Test III: Three truss assembly connected with 2x4 short member lateral restraint and diagonal bracing	4110	5256	3680	28	-11

Table 3. Buckling capacity of single and three truss systems.

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nection (Case 4), the average capacity was over 274 percent greater than the average capacity of Case 1.

The assembly considered here is relatively simple, but it emphasizes that *wood assemblies are structural systems that behave differently than single members.* System behavior cannot always be predicted with Euler's formula and in some cases Euler's formula (Eq. 1) may be conservative, and in other cases unconservative.

MPC wood truss assembly: As part of a separate truss testing program conducted at the truss testing facilities of Trussway, Ltd. in Houston the buckling behavior of the following three truss assemblies was examined:

- **Test I** – Measured the capacity for a single 24' Fink roof truss subject to a concentrated load near the peak.
- **Test II** – Measured the capacity of the truss evaluated in Test 1 subject to a concentrated load near the peak and connected by 2x4 short member lateral restraints to two adjacent trusses.
- **Test III** – Measured the capacity of the truss evaluated in Test 1 subject to a concentrated load near the peak and connected by 2x4 short member lateral restraints and diagonal bracing to two adjacent trusses.

The trusses evaluated in these tests were constructed by Trussway, Ltd. and had a 24-ft span with a 5-in-12 top chord slope. All the members in the trusses were nominal 2x4 Southern Pine 2400f-2.0E machine stress rated lumber connected at the joints with MiTek MII20, 20-gauge plates. The actual plate sizes used in these trusses were modeled explicitly in SAWFT. The capacity recorded for each test, together with the predicted capacities using the Euler equation and the SAWFT program are provided in Table 3.

Fig 3. Test II configuration of three 24-foot span Fink trusses interconnected with 2x4 short member lateral restraint.

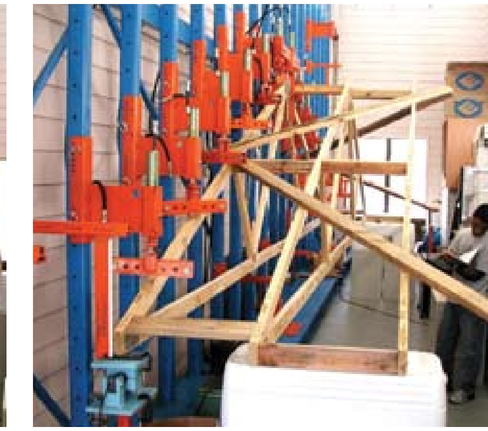

Fig 4. Test III configuration of three 24-foot Fink trusses with 2x4 short member lateral restraint and diagonal bracing.

Figure 3 shows the *Test II* configuration with the three trusses connected by 2x4 short member lateral restraint commonly referred to as spacer blocks. The spacer blocks were placed at the peak, heel and top chord panel point of each truss (see Figure 3) and attached with two 16d nails driven through the top chord and into the end grain of the spacer block. Figure 4 shows the *Test III* configuration with three trusses connected by spacer blocks and diagonal bracing. The diagonal bracing was attached to the top chords of each truss with two 16d nails.

The truss in *Test I* failed due to lateral buckling of the top chord. There was little or no damage to the member and plate connections so load was simply removed and the adjacent trusses and short member lateral restraints added for *Test II.*

Test II also resulted in lateral buckling of the top chord of the loaded truss after the nailed connection between the spacer block and the top chord failed due to end-grain withdrawal of the nails. Note that the capacity of this three truss assembly was nearly 76 percent greater than the capacity of the single truss.

Test III consisted of the same three trusses used in *Test II* with the addition of 2x4 diagonal bracing attached to the top chords (see Figure 4). Load was reapplied to the first truss in the assembly. Once again failure occurred when the top chord of the loaded truss buckled after end-grain nail withdrawal between the 2x4 lateral restraint and the truss heel.

The modulus of elasticity of each truss member was measured after the completion of *Test III* along with member and plate geometry and used to predict the peak load at the onset of instability using SAWFT. The predicted load values obtained using SAWFT are included in Table 3. For *Tests II and III,* the finite stiffnesses of the nailed supports were included in the analyses.

An attempt was also made to compute the Euler buckling load of the top chord of the loaded truss. Due to the amount of guess work involved in selecting the K_e value (Eq. 1) for Euler buckling, it is not surprising that the predicted load values derived using the Euler bucking equation vary considerably from the tested values. *It is unrealistic to expect a single-member equation to accurately predict a system response.*

Given that relatively few material properties were measured, the SAWFT predictions are surprisingly close to the instability loads obtained from the tests. For *Test I,* the SAWFT prediction was slightly conservative; for *Test II,* the SAWFT prediction was right on target; and finally, for *Test III,* SAWFT proved to be about 11 percent conservative. The Euler formula over predicted the capacity of the three truss assembly in *Test II and III* because of its inability to accurately model the impact of the lateral restraint and diagonal bracing members framing into the subject truss.

Discussion & Conclusions

The design of bracing for light frame wood structural systems continues to rely on Euler's formula and simple rules of thumb. Euler's formula is inherently limited because of its focus on single members. A variety of methods exist that incorporate the same matrix stiffness structural analysis methods already used in truss design to compute the instability of structural systems. This paper has briefly reviewed three methods and applied one of them to predict the instability of a simple truss system.

The method highlighted here employs a geometric stiffness matrix first developed by Yang and McGuire (1986). The method has been shown to be accurate as indicated by a series of single column and beam theoretical examples, and shows promise for predicting truss system instability loads. Examples of simple wood structural systems illustrated the impact that material variability can have on the instability of wood structural systems. These variabilities, such as the amount of initial member curvature and distribution of modulus of elasticity values can only be accounted for through the analysis of structural systems or large mechanical test programs. Although the verifications of the instability computation embodied in SAWFT are limited and preliminary, this work shows promise as a first step toward development of robust structural analysis algorithms for developing new design procedures for wood structural system stability.

For Acknowledgements and References, visit **Support Docs** at www.sbcmag.info. **SBC**

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